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ON THE TWO PHASE MODIFIED RATIO ESTIMATOR USING THE COEFFICIENT OF VARIATION OF AUXILIARY CHARACTERISTIC

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SUMMARY

A two phase ratio type estimator has been proposed using the coefficient of variation of auxiliary characteristics. The proposed estimator has superiority over the two phase modified ratio estimator proposed by Kwathekar and Ajagaonkar [2].

Keywords: Bias, Mean Squared Error, Coefficient of Variation, Auxiliary characteristics, Ratio estimator, Double sampling.

Introduction

For estimating the population mean $\overline{Y}$ of a certain chacteristics y, double sampling ratio estimator is well established in literature. It utilizes the information on auxiliary characteristic x which is highly correlated with y and is defined by

$$\bar{y}_{rd} = \bar{y}_n \, \bar{x}_{n'} / \bar{x}_n \tag{1}$$

where $\bar{y}_n$ and $\bar{x}_n$ are sample means of size n on characteristics y and x respectively. $\bar{x}_{n'}$ is the sample mean of character x, based on the preliminary large sample of size n'. Using C_x, the coefficient of variation of auxiliary characteristic x, Kwathekar and Ajagaonkar [2] proposed the estimator

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$$\bar{y}_{KA} = \bar{y}_n (\bar{x}_{n'} + C_x)/(\bar{x}_n + C_x) \tag{2}$$

which is more efficient than $\bar{y}_{rd}$ as well as $\bar{y}$ if

$$\bar{x}/2(\bar{x} + C_x) < K < (2\bar{x} + C_x)/2(\bar{x} + C_x), \tag{3}$$

where $K = \rho\, C_y/C_x$; C_y is the coefficient of variation of y and ρ is correlation coefficient between y and x.

Using the same information, we propose another two phase ratio type estimator defined by

$$\bar{y}_{Md} = \bar{y}_n\, \bar{x}_{n'} (1 + C_x)/(\bar{x}_n + \bar{x}_{n'} C_x). \tag{4}$$

2. Properties of the Proposed Estimator

Following Sukhatme *et al.* [3], it is easy to see that upto the first order of approximation, the proposed estimator will have bias

$$B\,(\bar{y}_{Md} = \bar{Y}\,\theta'\,(C_x'^2 - \rho\, Cy\, C_x') \tag{5}$$

and mean squared error

$$M\,(\bar{y}_{Md}) = \bar{Y}^2 \{\theta\, C_y^2 + \theta'\,(C_x'^2 - 2\,\rho\, C_y\, C_x')\}; \tag{6}$$

where $\theta = \dfrac{1}{n} - \dfrac{1}{N}$, $\theta' = \dfrac{1}{n} - \dfrac{1}{n'}$ and $C_x' = C_x/(1 + C_x)$.

For efficiency comparisons, we have

$$B\,(\bar{y}) = 0, \tag{7}$$

$$M\,(\bar{y}) = \theta\, \bar{Y}^2\, C_y^2, \tag{8}$$

$$B\,(\bar{y}_{rd}) = \theta'\, \bar{Y}\,(C_x^2 - \rho\, C_y\, C_x), \tag{9}$$

$$M\,(\bar{y}_{rd}) = \bar{Y}^2 \{\theta\, C_y^2 + \theta'\,(C_x^2 - 2\,\rho\, C_y\, C_x)\}, \tag{10}$$

$$B\,(\bar{y}_{KA}) = \theta'\, \bar{Y}\,(C_x''^2 - \rho\, C_y\, C_x) \tag{11}$$

and

$$M\,(\bar{y}_{KA}) = \bar{Y}^2 \{\theta\, C_y^2 + \theta'\,(C_x'' - 2\,\rho\, C_y\, C_x'')\}; \tag{12}$$

where $C'' = X C_x/(\bar{X} + C_x)$. It is easily verified that

$$B(\hat{y}_{Md}) < B(\hat{y}_{rd}) \text{ if } K < (1 + (1 + C_x)^{-1}) \qquad (13)$$

and

$$B(\hat{y}_{Md}) < B(\hat{y}_{KA}) \text{ if } K < \{\bar{X}/(\bar{X} + C_x) + (1 + C_x)^{-1}\}. \qquad (14)$$

It is also seen that

$$M(\hat{y}_{Md}) < M(\hat{y}) \text{ if } K > (2(1 + C_x))^{-1}, \qquad (15)$$

$$M(\hat{y}_{Md}) < M(\hat{y}_{rd}) \text{ if } K < (1 + (1 + C_x)^{-1})/2 \qquad (16)$$

and

$$M(\hat{y}_{Md}) < M(\hat{y}_{KA}) \text{ if } K < \{\bar{X}/(\bar{X} + C_x) + (1 + C_x)^{-1}\}/2. \qquad (17)$$

Thus the proposed estimator $\hat{y}_{Md}$ will be more efficient than $\hat{y}$, $\hat{y}_{rd}$ as well as $\hat{y}_{KA}$ if

$$\frac{1}{2(1 + C_x)} < K < \tfrac{1}{2}\left\{\frac{\bar{X}}{\bar{X} + C_x} + \frac{1}{1 + C_x}\right\}. \qquad (18)$$

3. Cost Aspect

Let C_1 and C_2 be the costs per unit of collecting observations on characteristics y and x respectively, then the total cost of survey apart from overhead cost can be expressed as

$$C_0 = C_1 n + C_2 n'. \qquad (19)$$

Minimizing (6) for the given cost C_0, the minimum mean squared error is obtained as

$$M_0(\hat{y}_{Md}) = \frac{S^2}{C_0} - \frac{\bar{Y}^2 C_y^2}{N} \qquad (20)$$

where $S = \sqrt{C_1}\, S_{y \cdot x} + \sqrt{C_2}\, S'_{y \cdot x}$,

$$S^2_{y \cdot x} = \bar{Y}^2 (C_y^2 + C_x'^2 - 2\rho C_y C^x)$$

and $S'^2_{y \cdot x} = \bar{Y}^2 (2\rho C_y C_x' - C_x'^2)$.

The optimal, in the sense of having minimum mean squared error, sample sizes are obtained as

$$n = \frac{C_0}{\sqrt{C_1}} \; \frac{S_{y \cdot x}}{S} \text{ and } n' = \frac{C_0}{\sqrt{C_2}} \; \frac{S'_{y.x}}{S} . \tag{21}$$

Minimum mean squared errors of the estimators $\bar{y}$, $\bar{y}_{rd}$ and $\bar{y}_{KA}$ for the above given cost are given by

$$M_0 (\bar{y}) = \left(\frac{C_1}{C_0} - \frac{1}{N} \right) \bar{Y}^2 C_y^2, \tag{22}$$

$$M_0 (\bar{y}_{rd}) = \frac{S_d^2}{C_0} - \frac{\bar{Y}^2 C_y^2}{N} . \tag{23}$$

$$\text{and } M_0 (\bar{y}_{KA}) = \frac{S_{KA}^2}{C_0} - \frac{\bar{Y}^2 C_y^2}{N} ; \tag{24}$$

where $S_d = \sqrt{C_1} \; S_{y.xd} + \sqrt{C_2} \; S'_{y.xd}$,

$$S_{y \cdot xd}^2 = \bar{Y}^2 (C_y^2 + C_x^2 - 2 \rho C_y C_x),$$

$$S_{y.xd}'^2 = \bar{Y}^2 (2 \rho C_y C_x - C_x^2)$$

and

$$S_{KA} = \sqrt{C_1} \; S_{y.xKA} + \sqrt{C_2} \; S'_{y \cdot xKA},$$

$$S_{y.xKA}^2 = \bar{X}^2 \; (C_y^2 + C_x''^2 - 2 \rho C_y C_x''),$$

$$S_{y.xKA}'^2 = \bar{Y}^2 (2 \rho C_y C_x'' - C_x''^2).$$

It is seen that

$$M_0 (\bar{y}_{Md}) < M (\bar{y}) \text{ if } \frac{C_2}{C_1} < \frac{S_y - S_{y \cdot x}}{S_y + S_{y \cdot x}}, \tag{25}$$

$$M_0 (\bar{y}_{Md}) < M (\bar{y}_{rd}) \text{ if } \frac{C_2}{C_1} < \left\{ \frac{S_{y \cdot xd} - R_{y \cdot x}}{S'_{y \cdot x} - S'_{y \cdot xd}} \right\}^2 \tag{26}$$

$$\text{and } M_0(\hat{y}_{Md}) < M_0(\hat{y}_{KA}) \text{ if } \frac{C_2}{C_1} < \left\{\frac{S_{y \cdot xKA} - S_{y \cdot xz}}{S'_{y \cdot x} - S'_{y \cdot xKA}}\right\}^2. \quad (27)$$

In deriving the conditions (25), (26) and (27) the conditions (15), (16) and (17) have been used respectively.

If the cost of collecting observation on auxiliary characteristic is cheaper, these conditions will obviously be satisfied. Thus the proposed estimator will have superiority over previous estimators under the condition (18).

4. Numerical Illustration

For the population considered on page 189 of Cochran [1], $N = 196$, $\bar{X} = 116$, $C_x^2 = 0.0156830$, $C_y^2 = 0.0142068$, $C_{xy} = 0.0146541$. $\rho = .9817412$ and $K = 0.9343939$. We assume that $C_0 =$ Rs 25.00, $C_1 =$ Rs 1.00 and $C_2 =$ Rs 0.10. The efficiency of the estimators has been computed in the following table :

Estimator	*n*	*n'*	*Absolute relative bias*	*Relative efficiency* (%)
$\hat{y}$	25	0	0	100
$\hat{y}_{rd}$	9	151	107.50×10^{-6}	601.89
$\hat{y}_k$	9	151	106.39×10^{-6}	602.85
$\hat{y}_{Md}$	9	153	66.59×10^{-6}	639.30

Thus the proposed estimator has smaller absolute bias and is more efficient than existing estimators.

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REFERENCES

[1] Cochran, W. G. (1977) : *Sampling Techniques*, Third ed. Wiley Eastern Limited.

[2] Kawathekar, D. M. and Ajagaonkar S. G. P. (1984) : A modified ratio estimator based on the coefficient of variation in double sampling. *J. Ind. Soc. Agr. Statist.* 36 (2) : 47-50.

[3] Sukhatme P. V., Sukhatme, B. V., Sukhatme, S. and Asok, C. (1984) : *Sampling Theory of Surveys with Applications*. New Delhi.